# MATH/QTM 385: Mathematics of Voting and Elections Vicki Powers

**Dept. of Mathematics Emory University**

This course looks at mathematical models of voting methods and procedures related to voting. The underlying theme is that of “fairness”, in the sense of the democratic ideal of reflecting the will of the voters. We look at properties of the models we study that relate to the idea of fairness. All of this is done in a mathematically rigorous and precise way, and this allows us to understand problems that can arise in elections and related procedures in real life, and possibly find better procedures. Past versions of the course have included sections on voting methods and on apportionment, which is the theory of dividing up resources fairly. We study apportionment in the context of dividing up the seats in the US House of Representatives among the states. I will add a new unit to the course on gerrymandering, which refers to drawing political boundaries in order to gain an advantage for some group, such as a political party. In recent years, mathematicians and others have studies methods for detecting and measuring gerrymandering with a view towards convincing courts that gerrymandering has taken place, using a mathematical model for redistricting (drawing political boundaries). This relates to sustainability in two ways. First, this is an issue of social justice since a gerrymandered districting plan will lead to representative bodies that do not reflect the make-up of the voters they are representing. Second, because of gerrymandering the representatives in the US federal legislature and in many state legislatures are more conservative than the voters who have elected them. This can lead to government policies that the majority of the voters do not agree with, for example we can see this with the issue of climate change where the voters would like the government to adapt policies that help mitigate climate change, but the government does not do so.

**Syllabus for MATH/QTM 385: Mathematics of Voting and Elections Instructor**: Professor Vicki Powers

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There is no textbook for this class. Notes on the topics discussed will be posted on Canvas.

**Mathematical background**. The mathematics used in this course is mainly logical deduc- tion and general quantitative reasoning along with limited use of probability and statistics. We will make use of some material from Math 250 Foundations of Mathematics, but since Math 250 is not a prerequisite for this course, this material will be presented in class. Some of the homework and group work problems will involve proof-type problems and one of the goals of this course is to develop skills in presenting a logical, carefully reasoned mathematical argument.

**Class Topics**. This class is about the mathematics of Social Choice Theory, which is the theory of group decision-making. There are three distinct topics that will be covered: Elec-

tion methods, the problem of apportionment (dividing up resources fairly) in the context of apportionment of seats in the U.S. House of Representatives, and the mathematics of gerrymandering.

**Grading**. Grades will be determined by homework (15%), quizzes (10%), group work in class (15%), a midterm exam (25%), and a final exam (35%).

**Exam**. There will be a midterm exam approximately halfway through the semester. I will announce the date of the midterm exam at least two weeks in advance.

**Quizzes**. There will be 5-6 short quizzes in class during the semester; quiz dates will be announced at least one week in advance.

**Homework**. The best way to learn mathematics is to do mathematics – doing your home- work carefully is very important in this class (and all mathematics classes). **Mathematics is not a spectator sport!** Homework will be assigned approximately every 2 weeks and collected the week after it is assigned. It will be due at the beginning of class and **no late assignments will be accepted.** There will be computational problems as well as proof-type problems. Please write neatly and in complete sentences where appropriate. **If I have difficulty reading your homework, you will receive points off or it will be returned without grading.**

**Group work**. We will spend a significant amount of time in class working on problems in groups. This is a great way to learn mathematics! Each group will hand in solutions at the end of class and these will be graded. Grades for the group work will be out of 10 points, as follows:

10 points - Excellent 9 points - Good effort

5 points - Minimal effort

0 points - Unexcused absence

I will set up groups once the drop/add period is over. The groups will consist of 3 people and each group will include at least one Math department major and at least one QSS major. If you would like to form your own group, let me know but be sure the group follows the rule of at least one major from each department.

**Plagiarism Policy**. When working on homework, you are free to consult sources (animate or inanimate) - working in groups is allowed. On the other hand, you are expected to make an honest attempt to do every problem on your own before consulting other sources. This is the only way you will learn the material of this course. Do not copy directly from any source, including another student’s paper. **Copying another student’s work or allowing your written work to be copied will be considered a violation of the honor code and will be treated as such.**

**Avoiding plagiarism.** A good rule of thumb to avoid plagiarism is the following: When doing the final write-up of a problem, do not have any text books, web pages, or classmate’s write-up in front of you. If you get stuck when writing up an assignment, go back and look again; just make sure that you organize the mathematics in your head before writing a solution rather than copying a solution from some source. **This is a generous homework policy. I will change the policy if I ftnd that students are abusing it.**

**Missing quizzes, the midterm exam, or group work.** No makeups will be given for a missed quiz or if you are absent from class during group work. If you miss class on a quiz day or a group work day for a legitimate, documented reason, let me know. For an excused absence from the midterm exam you will need a note from a dean. Note that putting a group member’s name on group work when they are absent will be considered a violation of the Honor Code and will be treated as such.

**Canvas**. There is a Canvas site for this class. Assignments, worksheets, and slides used in class can be found there. I will post a “daily diary” for each class day with a summary of what was done in class that day. I will enter your homework, quiz, and group work scores on the Canvas site. Please check occasionally that I entered your scores correctly and let me know immediately if there is a mistake.

**Electronics**. Do not use laptops, tablets, cell phones, etc. in class. Please turn them off and put them away.

**Honor Code**. *The honor code is in effect throughout the semester. By taking this course, you affirm that it is a violation of the honor code to cheat on exams, to plagiarize, to deviate from the instructions about collaboration on work that is submitted for grades, to give false information to a faculty member, and to undertake any other form of academic misconduct. You agree that the teacher is entitled to move you to another seat during examinations, without explanation. You also affirm that if you witness others violating the code you have a duty to report them to the honor council.*

**Schedule.** Below is a more detailed schedule of topics to be covered. Note this this schedule is tentative and subject to change.

## Social Choice

* 1. Voting systems – definitions, preference schedules, specific methods (1 week)
  2. Properties of voting systems (2 weeks)
  3. Ranking methods and Arrow’s Impossibility Theorem (1 week)

## Apportionment

* 1. Apportionment - definition and methods (2 weeks)
  2. The Balinski-Young Theorem - impossibility for apportionment (1 week)

## Gerrymandering

* 1. Introduction to redistricting and gerrymandering (1 week)
  2. Compactness and measures of compactness (2 weeks)
  3. The efficiency gap and related measures (1 week)
  4. Redistricting as a graph partition problem (1 week)
  5. Sampling methods (1 week)